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Project Network Models with Discounted Cash Flows
A Guided Tour through Recent Developments

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ABSTRACT

The vast majority of the project scheduling methodologies presented in the literature have been developed with the objective of minimizing the project duration subject to precedence and other constraints. In doing so, the financial aspects of project management are largely ignored. Recent efforts have taken into account discounted cash flows and have focused on the maximization of the net present value (*npv*) of the project as the more appropriate objective. In this paper we offer a guided tour through the important recent developments in the expanding field of research on deterministic and stochastic project network models with discounted cash flows. Subsequent to a close examination of the rationale behind the *npv* objective, we offer a taxonomy of the problems studied in the literature and critically review the major contributions. Proper attention is given to *npv* maximization models for the unconstrained scheduling problem with known cash flows, optimal and suboptimal scheduling procedures with various types of resource constraints, and the problem of determining both the timing and amount of payments.

(Project Scheduling - Discounted Cash Flows; Net Present Value)

1. Introduction

The vast majority of the project scheduling methodologies presented in the literature have been developed with the objective of minimizing the project duration subject to various types of precedence and resource constraints. For recent reviews, we refer the reader to the papers of Elmaghraby (1995) and Icmeli et al. (1993). In doing so, the financial aspects of project management are, unfortunately, largely ignored. When taken into consideration, there is a decided preference for the maximizaton of the net present value (*npv*) of the project as the more appropriate objective, and this preference increases with the project duration. Generally, a series of cash flows may occur over the course of a project in two forms. Cash outflows include expenditures for labor, equipment, materials, etc.. Cash inflows take place in the form of progress payments for completed work. The objective of this paper is to critically review the various contributions which try to capture the monetary and financial objectives of the project scheduling problem in the form of the maximization of the *npv*.

In recent years, a number of publications have dealt with the project scheduling problem under the *npv* objective. The majority of the contributions assume a completely *deterministic* project setting, in which all relevant problem data, including the various cash flows, are assumed known from the outset. Research efforts have led to optimal procedures for the *unconstrained* project scheduling problem, where activities are only subject to precedence constraints. In addition, numerous efforts aim at providing optimal or suboptimal solutions to the project scheduling problem under various types of resource *constraints*, using a rich variety of often confusing assumptions with respect to network representation (activity-on-the-node versus activity-on-the-arc), cash flow patterns (positive and/or negative, event oriented or activity based), and resource constraints (capital constrained, different resource types, materials considerations, time/cost trade-offs). A number of efforts focus on the simultaneous determination of both the amount and timing of payments. Last, a modest start has been taken in tackling the *stochastic* aspects of the scheduling problem involved.

The organization of the paper is as follows. In §2 we briefly discuss the various contract and payment structures and quickly review the rationale behind the *npv* objective. In §3 we introduce the basic problem types and assumptions. A guided critical tour of the expanding literature is offered in §4. §5 is then reserved for our overall conclusions and recommendations for future research.

2. Project networks and discounted cash flows

The idea of maximizing the net present value (*npv*) of the cash flows of a project as a concise and financially highly relevant criterion in deciding on the timing of activities in a project was introduced some twenty-five years ago (Russell 1970). The objective of this section is to examine the rationale behind this idea.

2.1 The npv criterion

The *npv* criterion lies at the very heart of capital budgeting and finance. Since the writings of Christenson (1955), Dean (1954) and Bierman and Smidt (1988) wise investment decisions are supposed to be based on a very simple principle. The value of an amount of money is a function of the time of receipt or disbursement of the cash. A dollar received today is more valuable than a dollar to be received in some future time period, because the dollar today can be invested to start earning interest immediately. The *accept-reject decision* of an (independent) project is then the result of a very simple mechanism. First choose an appropriate *discount rate* r (also called the *hurdle rate* or *opportunity cost of capital*), representing the return foregone by investing in the project rather than investing in securities. The *discount factor* $\beta = (1+r)^{-1}$ denotes the present value of a dollar to be received at the end of period *1* using a discount rate r. Second, estimate the future incremental cash flows on an after-tax basis and compute the net present value, *npv*, of the project using the formula:

$$npv = C_0 + \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t} \qquad [1]$$

where

C_0 = cash flow (usually a negative number representing the initial investment outlays) at the end of period *0* (that is, today)

C_t = cash flow at the end of period t

Sometimes Eq. [1] is replaced by its continuous equivalent assuming continuous discounting. The discount factor β is then simply replaced by $e^{-\alpha}$. The rule is then to accept the project if the *npv* is greater than or equal to zero and to reject it when the *npv* is less than zero.

Since it seems safe to assume that most project contractors have as their primary goal the maximization of their returns, not the least their financial returns, the expanding literature on project scheduling with discounted cash flows takes the fundamental view that it is appropriate not only to base the accept-reject decision on the *npv*-logic but also to *schedule* projects in order to accomplish some optimization of financial returns. As mentioned by Neo (1976) and March (1987), contractors have historically attempted to improve on the cash flow of their projects by *over-measurement* in the early months of the contract and *front-end loading* by artificially overpricing the activities to be done early in the project, and underpricing those that are to be completed later, while still maintaining the overall cost of the project. Basically, this tactic is an attempt to increase the value of a project by advancing the positive cash flows as much as possible.

The nature and timing of the cash flows generated by a project heavily depend on the contracts and on the payment structure used. In order to improve our understanding of the various assumptions used throughout the research efforts to be discussed, these are briefly reviewed in the next section.

2.2 Contracts and payment structures

Many different ways exist in which the **contract price** may be expressed or calculated. Though different practices do exist throughout countries and industries, a crucial distinguishing factor is whether the contract is fixed-price or not (Westney 1985,1992; Gilliard 1971; Twort 1986). The most common types of fixed-price contracts are bills-of-quantities contracts, price list contracts, schedule-of-rates contracts and lump-sum contracts. Among the contracts which are not set for a fixed price we distinguish between cost-plus percentage contracts, cost-plus fixed-fee contracts, target contracts and contracts with bonuses and penalties.

One of the most widely used fixed-price contracts is the *bills-of-quantities contract* (*contrat à bordereau de prix*). The total sum tendered under this type of contract is the sum of the individual items as priced in the bill, including any prime costs, lump sums and provisional sums. The quantities placed against the items showing the amount of work to be done are, for the purposes of tendering, quantities measured from the contract drawings. These quantities are measured as accurately as possible. When the work is actually done, the quantities are replaced by the measurement of the actual quantity of work the contractor carries out under each item. Again this is an accurate calculation. Owner and contractor commit on the basis of fixed unit prices but not on the quantities necessary for realizing the work. Twort (1986) lists the following superiorities of this method: (i) it results in payment to the contractor according to the amount of work done; (ii) it limits the price to be paid and if the work to be done is the same as shown on the contract drawings, then the owner pays exactly the tendered sum; (iii) the method gives freedom to alter the work and yet remains the basis of fair payment between owner and contractor; (iv) all tenderers price on exactly the same basis, and their tenders may therefore be closely compared with one another; (v) the bill itself gives every tenderer a very clear conception of the amount, kind and detail of work to be carried out. In short this type of contract is the most equitable type of agreement for both contractor and owner. However, there are a number of disadvantages: (i) prices have to be set on the basis of assumed quantities; (ii) the description of the items can be vague and/or incomplete, and (iii) the fixed price is not always linear in the quantity (overpricing of early work).

In industrial settings, large industrial concerns which are characterized by an almost constant rate of construction activities, usually provide the contractors with a very detailed *price list* for the various (standard) items. In order to put the contractors in sufficient competition, they ask the contractors to react to the price list (e.g. price list minus 7.5 %).

When it is not possible to foresee the full extent of work to be done (e.g. it is frequently not possible to state in advance how deep a borehole must be in order that it shall produce a given quantity of water), the contract can be based on a *schedule of rates*. In such a case, quantities against the individual items are either not inserted, or they are entered in estimated amounts or in round-figure provisional quantities. There is no implied guarantee given that all or any of the work scheduled will in fact be carried out. Therefore each item must carry its own overheads, and bring the contractor adequate reward if undertaken in large or small quantity, irrespective of the amount of work done under other items.

A *lump-sum contract* (*contrat à forfait*) consists of a single lump sum tendered and accepted as the fixed price. This type of contract works well provided the job is not very large, bears a small amount of risk (e.g. build a house or garage), and can be precisely described in all its details. Quite often, a large bill-of-quantities contract may contain within it single items which are in effect lump-sum contracts for portions of the work within the overall contract. Lump-sum contracts give the owner the assurance of a fixed total price and avoid a lot of detailed accounting and measurement work, but they immediately run into trouble if the owner wants an alteration of design, or if the job itself runs into unforeseen troubles. Sometimes, lump-sum contracts are used in conjunction with a schedule of rates which are to be applied to the pricing of variations (*contrat à forfait relatif*).

Cost-plus contracts come in two forms. *Cost-plus percentage contracts* (*contrats en régie*) give no assurance of limitation to the total cost. In this rather impopular contract, the contractor is paid the actual expenditure (direct cost) incurred in the purchase of materials, employment of labor and plant, and he is paid a percentage over and above this to reimburse him his overhead expenses and profit. Contractors do not make any commitments, nor on quantitites nor on unit prices. Owners do not like this contract because it gives no incentive to the contractor to be efficient (the less efficient the contractor, the higher the cost and profit). Contractors do not like it because every document needs to go through a though check before it can finally be authorised for payment. This type of contract is used only in an emergency, for a limited period, before there has been suffficient time to draw up another form of contract. *Cost-plus fixed-fee contracts* (*contrats en régie plafonnés*) also require that the contractor is paid his actual costs, but the fee which is intended to cover his overheads and profit is fixed. The fixed fee may be tendered in competition with other contractors, or it may be negotiated between owner and contractor.

Target contracts (*contrats en régie avec intéressement*) are much like cost-plus fixed-fee contracts, but the fee (or profit) to the contractor increases if the final cost of the work is less than the estimate and decreases if the final cost is more. If the actual costs, A, are less than the target costs, Q, the contractor is paid $A+(Q-A)E+F$, where F is the fee or profit of the contractor. If A exceeds Q, the contractor is paid $A-(A-Q)E'+F$. The coefficients E and E' are agreed on the outset and are set between 0 and 1 (usually between 0.25 and 0.5). If time is particularly vital, it is possible to build in an additional incentive by varying the share of the savings accrueing to the contractor according to the extent to which the contract is completed early or late. In a way, target contracts hold a kind of contradiction. If all the operations of a job to be done can be specified in advance, there is no reason to have a target at all and a bill-of-quantities contract can be used. On the other hand if operations cannot be foreseen, or in situations of high risk, it is often practically impossible to set the targets.

Bonuses and penalties can be included in any sort of contract if it relates to completion of the whole or part of the work within a given time, provided a practicable time target is set. In addition, performance bonuses or penalties may be related to the output or efficiency of the finished works. The purpose of the use of bonuses and penalties is to align the objectives of owner and contractor by giving the contractor a profit incentive to do what also benefits the owner. A good incentive plan should be designed so that it is quite possible for the contractor to earn a bonus. The benefits to the owner should be such that he wants to pay the bonus as the cost of the bonus is far less than the financial benefit he derives from improved contractor performance (Westney 1985). There are two general categories of incentive plans. *Unilateral* incentive plans are not negotiated: the owner simply makes the contractor aware that he will receive a certain bonus if he meets certain targets. For example, he might offer a bonus for completion on-schedule, with an increased bonus for every day that completion is ahead of schedule. It is a simple plan in that it is a "take it or leave it offer". As a result, such a plan usually provides a bonus for good performance, but no penalties if performance is not up to expectations. *Bilateral* incentive plans are negotiated: the owner and contractor agree on every aspect of the plan and administer it in close cooperation. This type of plan can involve benefits for good performance as well as penalties for bad performance. The incentive plan must be carefully designed and administered such that it is difficult but achievable. The benefit is clearly lost if it is too easy. If the target is too difficult, a lot of money and effort is wasted in pursuit of an impossible goal.

It is evident that the various contract types discussed above are not mutually exclusive. Certain items may be on a lump-sum basis whilst others may be subject to remeasurement, etc.. The precise contract specifications will differ among countries and industries, and will depend largely on the amount of information regarding the job to be done and the conditions under which it will be carried out.

The terms of **payment** may also be the subject of different types of policies. The owner may attain the best results if he offers the tenderers terms of payment which, while providing him with reasonable contractual safeguards, impose the minimum strain on the contractor's financial resources. In doing so, the owner will (a) avoid having to restrict the tenderer list to large firms possessing the resources to finance the contract, (b) ensure that the tenderers do not have to inflate their tender prices by financing charges (in many instances the rate of interest which the contractor has to pay when borrowing will be higher than that paid by the owner), (c) minimise the risk of having to work with a contractor who has insufficient cash (Marsh 1987). On the other hand, in doing so, the owner has to finance the work in progress and tie up his own capital in advance of obtaining any return on his investment.

Expenditure patterns will be different for different types of costs. If the work is done on a reimbursable basis (as in cost-plus contracts), labor expenditures (engineering, direct and indirect labor) tend to be linear over time as an invoice is usually submitted periodically (monthly) for work done during the previous period. Overhead costs (field and office overheads) will also generally follow a linear expenditure pattern over time. They may be directly reimbursed, or included in the hourly rate. Materials are usually invoiced when delivered, fabricated materials may require progress payments. The expenditure pattern for materials tends to be more like a point or series of points. In all cases, it is reasonable to assume that there is an elapsed time between the receipt of an invoice and the payment (in most cases, a 30-day period). In addition, the timing of the amounts to be paid depends on the specific contract used. Lump-sum contracts specify that the lump sum may be paid in full upon completion of the work, but most often lump sums are paid in increments according to progress. Reimbursable contracts usually have payments to be made periodically for work performed. This can be done on the basis of certificates issued by the project engineer. Retention money may be involved, e.g. 5 per cent to be released on the issue of the certificate of practical completion and 5 per cent at the end of the defects liability period. In practice, the final amount is not settled until a number of months after the end of the contract and includes settlement of claims (Marsh 1987). Obviously, if the work is being controlled through network analysis, values can be allocated to certain key activities, and the contract can provide that payment of these sums will be made as those activities are completed.

3. Problem types and assumptions

Many formulations for the project scheduling problem - essentially the problem of determining the starting time (completion time) of the project activities - have been offered throughout the literature. The formulations differ in both the type of objective function used and the different types of constraints. In the basic deterministic project scheduling problem the only constraints explicitly taken into consideration are the precedence constraints among the activities which are usually of the finish-start type with a time lag of zero; i.e., an activity can start as soon as all its predecessor activities are finished. The most commonly used objective function used for this problem setting is the minimization of the overall project duration (project makespan). Activity durations are usually assumed to be known and integer and activity preemption is not allowed (activities have to be completed once started). The solution to this *unconstrained min-duration problem* (unconstrained in the sense that resource availabilities are infinite) is obviously the longest (critical) path in the network. When the min-duration objective is replaced with an objective function that attempts to maximize the net present value (*npv*) of cash flows ensuing from the project schedule, we have the *unconstrained max-npv problem* (it should be noted from the outset that authors such as Grinold (1972) denote the unconstrained problem as the 'payment scheduling problem'; as indicated below, we prefer to associate that terminology with the case where both the amount and timing of cash flows need to be determined). In addition to the assumptions mentioned above, most formulations assume that cash flows are known in both their amounts and timing. Cash flows can be associated with certain events (milestones) in the (activity-on-the-arc) project network; cash flows can be compounded to the end or discounted to the beginning of project activities; or cash flows may not occur at events but at regular periods (e.g. months). Models may assume a mixture of positive and negative cash flows, may assume only positive cash flows or a single positive cash flow at project completion. In addition to the cash flows related to project activities, models may incur overhead costs at each period before the project gets completed. Projects may be scheduled with or without due dates and bonuses and penalties may be imposed on certain events (usually the event which marks the completion of the project). As will be shown below, the unconstrained max-npv problem accepts optimal solutions, such that there is no need for heuristics. Despite the fact that the introduction of the *npv* objective yields a nonlinear programming problem, the principle underlying a solution to the *deterministic problem* is essentially simple: positive cash flows should be advanced as much as possible, while negative cash flows should be the subject of maximal delays. Optimal schedules that result from the max-npv objective function, however, will not necessarily be the same as the min-duration schedule. As will turn out, the *stochastic* unconstrained max-npv problem is a very hard nut to crack. When both the amount and timing of the cash flows must be determined, we have what we call in this paper, the *payment scheduling problem*, for which only a few models have been developed so far.

As in the case of the min-duration problem, imposing limited resource availabilities upon the max-npv problem will result in the NP-hard *resource-constrained max-npv problem.* Various types of resource constraints may be accommodated in the models. *Renewable resources* are those whose availabilities are fixed for a certain period and are replenished as a new period is started. An example is the number of workers available per day. *Nonrenewable resources* are not renewed once used. An example is a limited supply of material or capital available for the entire project. *Doubly constrained* resources have constraints on both total usage and total consumption. Sometimes special attention is given to capital availability constraints.

When capital availability is treated on a per-period basis, where the capital available for use per period is fixed and the cash requirement of each activity is defined as a constant amount for each period in which that activity is active - and if it is assumed that all cash inflows go into a central pool and will not be reinvested in the project that generated them - then cash may be added as another set of renewable resource constraints. Often, however, capital is treated as a limited non-renewable resource which is reduced by cash outlays and which is boosted by cash inflows. In this *capital constrained max-npv problem*, at any time period during the project, the capital available will be the result of the cumulative effect of all scheduling decisions made since the start of the project. Whereas the basic capital constrained max-npv problem assumes that cash availability is fixed, in many projects the project capital may be augmented by borrowing capital from an external entity up to a specified limit (line of borrowing). In addition to capital constraints, authors have paid attention to the acquisition of different types of materials resulting in various types of *inventory balance constraints*.

Sometimes the duration of an activity is a function of the amount of resources committed to it. The resulting *max-npv time/cost trade-off problem* then involves the simultaneous determination of activity starting times and durations. When the time/cost trade-offs are discrete, i.e. activities can be performed in multiple modes, we have the so-called *multi-mode max-npv problem.* The next section aims at a critical review of the literature on project scheduling with discounted cash flows. Table 1 provides a basic classification.

4. A guided tour through the literature

4.1 The unconstrained max-npv problem

4.1.1 Deterministic models

To the best of our knowledge, *Russell* (1970) was the first to introduce the idea of maximizing the net present value of the cash flows in a project. He deals with the *unconstrained max-npv problem* by taking an *event-based* view in which both positive and negative cash flows occur as events in the project are completed. Consider a project with m activities (m arcs in the activity-on-arrow mode of representation) with fixed durations $\{d_k\}$ (k=1,...,m), and n events (n nodes in the activity-on-the-arc mode of representation), to occur at time instants $\{T_i\}$, with associated net cash flows $\{C_i\}$ (i=1,2,...,n). Russell's objective function then is to

$$\text{maximize} \quad \sum_{i=1}^{n} C_i \; e^{-\alpha T_i} \qquad [2]$$

	Event-oriented		*Activity-oriented*		
	Optimal	*Suboptimal*	*Progress payments*		*Payment only at the end*
			Optimal	Suboptimal	
Deterministic unconstrained max-npv	Russell (1970) Grinold (1972) Elmaghraby & Herroelen (1990) Herroelen & Gallens (1993)	no research needed	Sepil & Kazaz (1994)		Smith-Daniels (1986)
Stochastic unconstrained max-npv					Buss & Rosenblatt (1993)
Resource-constrained max-npv					
renewable resources		Russell (1986) Padman et al. (1990) Padman & Smith-Daniels (1993a) Ulusoy & Özdamar (1994a) Zhu & Padman (1993) Padman & Zhu (1994)	Yang et al. (1992) Baroum (1992) Icmeli & Erengüç (1995)	Sepil & Ortaç (1995) Ulusoy & Özdamar (1995) Baroum & Patterson (1993) Icmeli & Erengüç (1994) Yang et al. (1995) Smith-Daniels & Aquilano (1987)	
nonrenewable resources		Padman & Smith-Daniels (1993b)	Doersch & Patterson (1977) Smith-Daniels[2] (1987) Patterson et al. (1990)		
doubly-constrained		Ulusoy & Özdamar (1994b)			
Payment scheduling problem	Dayanand & Padman (1993a) Dayanand & Padman (1993b)		Dayanand & Padman (1993a)		

Table 1. Classification of the max-npv literature

where $e^{-\alpha} = 1/(1+r) = \beta$, the discount factor. For uniformity of expression, the criterion [2] is sometimes re-written as:

$$\text{maximize} \quad \sum_{i=1}^{n} C_i \beta^{T_i} \qquad [2']$$

This maximization is subject to the time precedence constraints

$$T_{i(k)} \quad + \quad d_k \quad \leq \quad T_{j(k)} \quad , k = 1, \ldots, m \qquad [3]$$

where *i(k)* and *j(k)* denote the tail and head nodes of activity *k*, respectively. Clearly, Eq. [2] (or [2']) represents the maximization of the *npv* of the project, while Eqs. [3] represent the enforcement of the precedence relationships on the network activities.

The nonlinear programming problem of Eqs. [2]-[3] is solved by Russell iteratively through successive approximation in the following manner. Initially, the nonlinear objective function Eq. [2] is approximated by considering only the first (linear) term of the associated Taylor series expansion. Assuming a current non-optimum but feasible solution given by the event times T_i^o, we have, for T_i close to T_i^o,

$$\sum_i C_i e^{-\alpha T_i} \approx \sum_i C_i e^{-\alpha T_i^o} - \sum_i (T_i - T_i^o) C_i \alpha \ e^{-\alpha T_i^o}$$

$$= \sum_i C_i e^{-\alpha T_i^o} + \sum_i T_i^o C_i \alpha e^{-\alpha T_i^o} - \sum_i T_i C_i \alpha e^{-\alpha T_i^o}$$

$$= const - \sum_i T_i C_i \alpha e^{-\alpha T_i^o}$$

and the original objective Eq. [2] is replaced by the maximization of the linear objective:

$$-\sum_i T_i \ C_i \alpha \ e^{-\alpha T_i^o} = -\sum_i T_i C_i \alpha \beta^{T_i^o},$$

subject to the precedence constraints Eqs. [3].

The dual form of this (primal) linear programming (LP) model turns out to be a transshipment problem over a network model. The solution of this transshipment model yields a system of flows. By the complimentary slackness principle of LP, flows will only occur in arcs whose corresponding primal activity has no float. As such, the flows on the arcs of the transshipment model impute an occurrence time for each node of the network. These imputed event times are then utilized in a Taylor series expansion to provide an improved linear approximation to the primal nonlinear objective function. The parameters of the associated dual LP are subsequently updated and the solution of the updated (with new times) transshipment model yields a new set of event realization times. The whole process is repeated until successive event times at all nodes are identical. Russell provides proof that the resulting node times do converge, and their point of accumulation constitutes at least a local optimum of the original *npv* maximization problem. Apart from an example illustrating the application of the algorithm, he does not report any computational experience.

Russell's activity-on-the-arc network example appears in Figure 1. The numbers adjacent to the arcs denote the fixed activity durations. The numbers adjacent to the nodes represent the cash flows associated with the corresponding events. This example was used by Elmaghraby & Herroelen (1990) to reveal the essential 'simplicity' of the *unconstrained max-npv problem.* The separability of the objective function (Eq. [2] or Eq. [2']) in the times of realization $\{T_i\}$ of the various events, combined with the fact that the discount factor β ($\equiv e^{-\alpha}$) is smaller than one, lead one to conclude that the optimal value of T_i is determined by the sign of its coefficient a_i : if positive, then T_i should be as small as possible (thus making its term β^{T_i} as large as possible), and if negative, then T_i should be as large as possible (thus making its term β^{T_i} as small as possible). How small or large T_i can be is determined, of course, by the precedence constraints Eq. [3]. Viewed from this perspective, the scheduling problem reduces to the problem of either advancing some node realizations or retarding them while respecting the precedence relations.

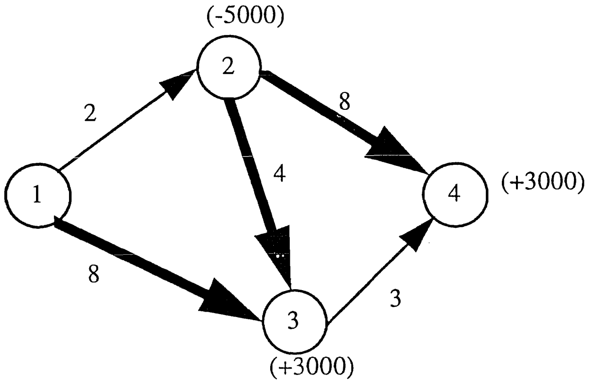

Figure 1. Example network

Referring to Figure 1, we have that $a_2 < 0$ which makes T_2 as large as possible. Since a_3 and a_4 are both > 0, the realization times T_3 and T_4 should be as small as possible. Assuming the project starts at time $T_1=0$, it is shown by Elmaghraby and Herroelen (1990) that under these conditions, the precedence constraints of Eq. [3] result in the tree shown in heavy lines in the figure. Given the cash flows and $\beta = e^{-.01} \approx .99$, the value of the objective function is maximized for $T_2^* = 4$, $T_3^* = 8$ and $T_4^* = 12$. This leads to a *npv* of 626. As can be seen, the *duration of the optimal schedule is longer than the critical path of the min-duration schedule* ($T_2^* = 2, T_3^* = 8, T_4^* = 11$), which has a *npv* of only 554! The fact that schedules which maximize the *npv* may have a longer duration than minimum duration schedules has also been observed by Bey et al. (1981). They also observe that even in the case where the optimal *npv* schedule achieves a duration equivalent to an optimal min-duration schedule, alternative min-duration schedules (achieved by feasible reassignments of nonterminal activities) will result in suboptimal *npv*'s.

Grinold (1972) also takes an event-based view and shows that the nonlinear program with linear constraints and a convex objective function of Eqs. [2]-[3] can be transformed into an equivalent linear program (Grinold's treatment is equally valid when applied to non-concave objective functions). This fact is then used to demonstrate that the optimal solution of the scheduling problem corresponds to a feasible tree in the project diagram which consists of all arcs having no float; i.e., an extreme point in the set of feasible schedules. As a consequence, Grinold restricts the search for optimal schedules to feasible trees in the project network. Using standard complementary slackness results for checking the optimality of the feasible trees, he develops two solution procedures which are related to Markowitz's special procedure for the weighted distribution problem, requiring the solution of triangular systems of equations with all matrix coefficients equal to ± 1 or 0. The first algorithm solves the problem for a fixed *project deadline*. The second, parametric algorithm solves the problem for all possible project deadlines. This yields a curve explicitly showing the trade-off in project duration and present value. Again, apart from an example illustrating these computations, no further computational results are given.

Elmaghraby and Herroelen (1990) argue that the approaches of Russell (1970) and Grinold (1972) may yield inconclusive results following from the fact that, in the absence of due dates, the optimal schedule may be to delay the project for ever, and it may happen that any schedule is optimal. Under these conditions, the proposed LP's and the iterative procedure based on them suggested by both authors shall either fail to identify the result, unrealistic as it might be, or cycle forever without yielding a definite answer.

Elmaghraby and Herroelen (1990) have encapsulated the above mentioned intuitive arguments (schedule positive cash flows as early as possible, and negative cash flows as late as possible) into an intuitively appealing algorithm. The algorithm operates by building tree structures in an activity-on-the-arc network in an iterative fashion, and by determining proper displacement intervals for the trees. *Herroelen and Gallens* (1993) streamline the event-based algorithm and report on favorable results obtained on 250 randomly generated projects with a computer code written in the C language and running under the DOS operating system. Sepil (1994) reports on a possible flaw in the algorithm based on its description in Elmaghraby and Herroelen (1990). Proper implementation of the code described by Herroelen and Gallens (1993), however, avoids the reported flaw. Our conclusion from the discussion so far can be that the unconstrained max-npv problem can be solved in an efficient and optimal manner, so that there is no need for the use of heuristics.

In a recent working paper, *Sepil and Kazaz* (1994) study the unconstrained max-npv problem under different assumptions. Instead of associating negative and/or positive cash flows with events in an activity-on-the-arc network they aim at large scale projects, especially in the construction sector, where cost-plus types of contracts specify that payments (estimated costs augmented by a profit margin) are to be made at the end of regular periods (months) for the finished and the partially finished activities during the period. Assuming that the costs of activities occur at the activity completion times, they derive the per period activity cost by dividing the cost of the activity by its duration. They formulate the problem as an integer programming problem using so-called activity profit curves which show how the *npv* of cash flows associated with the activity change with respect to activity finish times, and which are approximated by piecewise linear functions.

Smith-Daniels (1986) attempts to build a model to predict the *npv* of a restricted category of the unconstrained max-npv problem where the only positive cash flow occurs at completion of the project (such as in projects under lump sum contracts). Given the simple logic clarified earlier, it is obvious that under the prerequisite of a positive *npv* and a critical-path based due date, activities falling on the critical path must be assigned to their critical path determined early start times, while those off the critical path must be assigned to their late start times. Smith-Daniels develops several summary measures to predict the project *npv* in this specific problem.

4.1.2 Stochastic models

The only contribution on the extremely complex *stochastic unconstrained max-npv problem* we are aware of is due to *Buss and Rosenblatt* (1993). The authors assume activity-on-the-arc networks in which activity durations are exponentially distributed with known mean and variance. Upon completion of an activity i a cost a_i is incurred (this assumption can be relaxed to more general types of cash flow, including progress payments) and upon completion of the project a single revenue R is received. They aim at determining the optimal amount of activities' delay, beyond their earliest start times, so as to maximize the expected net present value of the project. They obtain results on the *static* problem of delaying one activity, and analyze the problem of delaying several activities at once only for situations in which the delayed activities are in strict precedence relationship. Both optimal and suboptimal procedures are provided to maximize the expected net present value with respect to the amount of delay. They test the robustness of their results by simulating projects having symmetric and asymmetric Beta probability distributions.

The added complexity of the stochastic problem over the deterministic one is largely due to the fact that the simple logic of always delaying activities with negative cash flows as much as possible no longer applies. Delaying slack activities (with slack defined by the usual CPM analysis and activity durations taken at their *mean* values) with associated negative cash flow may result in delaying the completion of the entire project and its associated positive cash flow, which may result in a net decrease in present value. This can be illustrated on the two-activity project example represented in Figure 2 and borrowed from Buss and Rosenblatt (1993). The duration of activity 1 is exponentially distributed with a mean $1/\mu_1 = 2$ months and a cost of $a_1 = \$15,000$. Activity 2 also has an exponential distribution with mean $1/\mu_2 = 20$ months and cost $a_2 = \$10,000$. The revenue at the end of the project is R = \$30,000. The discount rate is 0.01 per month. If the activities are deterministic the *npv* for the early-start schedule is \$1671.63, whereas the *npv* of the late-start schedule is \$4093.65. However, if the activities' durations are independent and exponentially distributed with the parameters specified, we have an expected npv=\$1908.26 for the early-start schedule and an expected npv=\$2536.61 for the optimal delay schedule in which activity 1 is delayed by 12 months. If the cost of activity 1 is increased to \$17,400, then for the deterministic case the present values are -\$680.84 for the early start schedule and \$2128.70 for the late start schedule. For the stochastic case, we have an expected npv = -\$444.68 for the early start schedule and \$484.51 for the optimal delay schedule in which activity 1 is delayed by 15.5 months. Thus, we see that for both deterministic and stochastic projects, delay can be used to make an unattractive project attractive. Note also that in this case the amount of improvement is substantially less for the stochastic network.

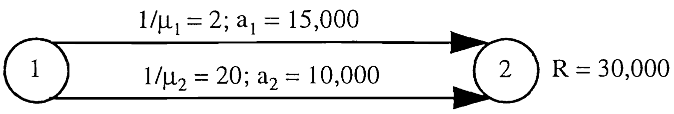

Figure 2. Stochastic project example

Whereas a full treatment of the unconstrained max-npv problem requires taking into account the uncertainty in the durations of the activities, this is no minor feat. In recent years, applicability of the orthodox *npv* theory for making capital budgeting decisions under uncertainty has been seriously questioned (Dixit and Pindyck 1994). The *npv* rule assumes that either an investment is reversible, that is, it can somehow be undone and the expenditures recovered should market conditions turn out to be worse than anticipated, or, if the investment is irreversible, it is a now or never proposition, that is, if the firm does not undertake the investment now, it will not be able to in the future. However, the ability to *delay an irreversible investment expenditure* cannot only profoundly affect the decision to invest, but also undermines the simple *npv* rule. The reason is that a firm with an opportunity to invest is holding an "option" analogous to a financial call option - it has the right but not the obligation to buy an asset at some future time of its choosing. When a firm makes an irreversible investment expenditure, it kills its option to invest by giving up the possibility of waiting for new information to arrive that might affect the *desirability* or *timing* of the expenditure. This lost option value is an opportunity cost that must be included as part of the cost of the investment. As a result the *npv* rule 'invest when the value of a unit of capital is at least as large as its purchase and installation cost' must be modified. The value of the unit must exceed the purchase and installation cost, by an amount equal to the value of keeping the investment option alive. Dixit and Pindyck (1994) develop the basic theory of irreversible investment under uncertainty, emphasizing the option-like characteristics of investment opportunities. Translating this idea to the context of project scheduling under uncertainty is a tempting area for further research.

4.2 The deterministic resource-constrained max-npv problem

Adding renewable resource constraints to the model of Eqs. [2]-[3] yields the NP-hard (Baroum 1992) *resource-constrained max-npv problem.* Conceptually, the resource constraints on renewable resources take the following form:

$$\sum_{i \in S_t} r_{ik} \leq b_k, \quad t = 1,2,\ldots,f_n, \quad k = 1,2,\ldots,K \qquad [4]$$

where r_{ik} is the amount of renewable resource type k required by activity i, b_k is the total availability of resource type k, f_n is a decision variable denoting the finish time of the single end node of the project and $S_t = \{i: f_i - d_i < t \leq f_i\}$, with d_i as the fixed activity duration, is the set of activities in progress in the time interval]t-1,t].

Figure 3 repeats the problem example of Figure 1 with the additional data on the resource requirements for two types of renewable resources (indicated between parentheses along the arcs), the constant availability of which is set to 3 units each. The optimal solution yields an npv = \$548 with $T_2^* = 4$, $T_3^* = 8$ and $T_4^* = 15$.

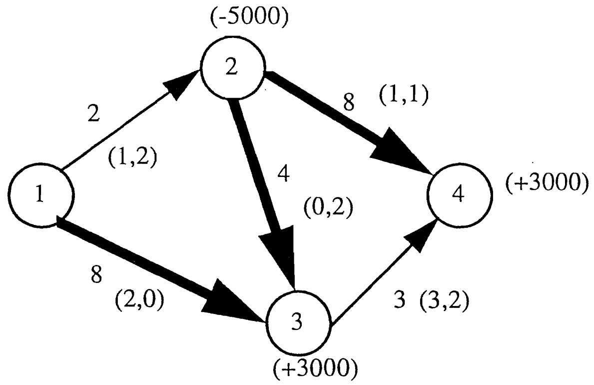

Figure 3. Resource-constrained problem example (Russell 1986)

Various optimal and suboptimal procedures for the resource-constrained max-npv problem have been presented in the literature. They widely differ in their assumptions.

4.2.1 Optimal procedures

Yang, Talbot and Patterson (1992) describe an integer programming algorithm for the resource-constrained max-npv problem in activity-on-the-node networks with the assumption that the start of an activity requires an initial capital investment that is recovered upon completion of the activity. The authors take an activity-based view in assuming that cash flows (cash payments and cash disbursements) occur during the performance of each activity. A value at completion is determined for an activity by compounding capital requirements and associated cash flows to the end of the activity. In other words, the value of an activity upon completion is given by

$$D_j = \sum_{t=1}^{d_j} F_{jt}\, e^{\alpha(d_j-t)} + C_j\,(1-e^{\alpha d_j}) \qquad [5]$$

where

D_j = terminal value of cash flows in activity j at its completion

F_{jt} = cash flows for activity j in period t, $t=1,2,...,d_j$

$e^{-\alpha}$ = discount factor

d_j = duration of activity j

C_j = capital investment required by activity j

The first term in Eq. [5] gives the sum of the cash flows associated with an activity (F_{jt}) times the appreciation factor to compound their values to the end of the activity $(e^{\alpha(d_j-t)})$. The term $(d_j\text{-}t)$ gives the number of periods under which the cash flow occurring at the end of period t has to be discounted. The second term in Eq. [5] gives the value of the opportunity cost incurred by holding the initial investment (C_j) for the duration of the activity.

Allowing for bonuses (penalties) upon project completion, the objective function used can be conceptually written as follows:

$$\max \quad \sum_{j=1}^{n} q_{f_j} D_j \quad + \quad q_{f_n} B_{f_n} \qquad [6]$$

where

q_t = factor for discounting over t periods to time 0

B_t = bonus for completion of the project at time t ($B_t < 0$ implies that B_t is a penalty)

f_j = an integer variable representing the finish time of activity j ($f_n \leq$ project due date)

The authors have modified the implicit enumeration algorithm by Talbot and Patterson (1978) for the resource-constrained min-duration problem. The fathoming technique employed in the original algorithm (the network cut) to eliminate inferior partial schedules is successfully modified to accommodate the *npv* based objective. Yang et al. (1992) report on computational results on a set of 10 problems (from 6 to 21 tasks) with a project due date set equal to one period greater than the minimum resource-constrained duration. The amount of computation time required is heavily dependent on the due date and increases significantly when the potential schedule duration increases well beyond the minimum resource-constrained duration.

Baroum (1992) addresses a special case of the resource-constrained max-npv problem where only activities having positive *npv*'s (that is positive D_j in Eq. [5]) are considered in completing the project. This is usually the case in cost-reimbursement contracts such as cost-plus percentage-fee contracts. The procedure is an extension of the very efficient depth-first *DH-procedure*, originally developed by Demeulemeester and Herroelen (1992) for the resource-constrained min-duration problem. The unique characteristics of the problem under study (positive *npv*'s) are effectively exploited to reduce the search effort as he considers only semi-active partial schedules where each activity is started as soon as possible within precedence and resource constraints. Branching only occurs in order to resolve resource conflicts by delaying subsets of activities. The delay alternative which generates the highest *npv* upper bound is selected for branching. Two fathoming rules are used: left-shift dominance and comparison pruning. Because only activities with positive *npv*'s are considered, any schedule that is left-shift dominated is also *npv* dominated. Once a partial schedule has been proven to lead to a reduced solution space, two fathoming rules may be applied. The first rule recognizes that when the *npv* contribution of the scheduled activities in a current partial schedule which has a smaller solution space than a previously saved partial schedule is less than the *npv* contribution of the corresponding activities in the saved partial schedule, the current partial schedule is inferior. The second rule is based on the computation of an upper bound for the current partial schedule (obtained by generating the optimal resource-relaxed schedule resulting from augmenting the project network by the delay arcs corresponding to the selected delay alternative) and comparing it to the *npv* of the incumbent solution. Baroum (1992) reports on computational results obtained on the 110 test problems (subsequently referred to as the Patterson problems) originally assembled by Patterson (1984) for the resource-constrained min-duration problem. The procedure was able to solve under a variety of activity cash flow distributions all problems within 7 seconds on an IBM RISC System/6000, leading the author to conclude that the algorithm is quite practical for solving projects having fewer than 50 activities.

Icmeli and Erengüç (1995) present a branch-and-bound procedure for the resource-constrained max-npv problem. They assume activity-on-the-node networks where the cash flows associated with the project activities are compounded to their completion time according to the first part of Eq. [5]. Contrary to Baroum (1992) they allow for positive and negative cash flows. The project due date is obtained as $s*D$, where D is the project duration obtained from a heuristic solution procedure described by Icmeli and Erengüç (1994), and s is a constant greater than 1. The branch-and-bound procedure is also to be considered an extension of the *DH-procedure* (Demeulemeester and Herroelen 1992) for solving the resource-constrained min-duration problem. At each node of the search tree a complete schedule which may be resource infeasible is obtained. At the initial node of the tree an optimal solution to the corresponding unconstrained max-npv problem is obtained using the fixed deadline algorithm of Grinold (1972), yielding an upper bound. If this solution is resource feasible the procedure terminates. If not, branching is done using the minimum delaying alternatives concept to resolve resource conflicts. Resource conflicts are resolved by adding extra precedence constraints to the unconstrained problem. The subproblems thus obtained are solved using Grinold (1972). A node is fathomed either if the optimum unconstrained solution has a project duration exceeding the due date, or if it is less than or equal to that of the incumbent solution. The node with the greatest objective function value is selected for further branching. The algorithm is written in Fortran and run on an IBM3090 computer with vector processing. In implementing the algorithm, the authors adopted tolerance levels which guarantee that the solution value obtained by the algorithm is within $(100\in)\%$ of that of the optimal solution, with $\in$ ranging from 0 to 0.05. The computational experiment used 50 problems taken from Patterson (1984) with cash flows generated randomly from a uniform distribution on [-500,1000], and 40 problems (32 activities, 3 resource types) generated using the *Progen* generator (Kolisch et al. 1992) with cash flows generated from the uniform distribution on [-5000,10000]. Using 0% tolerance, 34 problems could be solved with a CPU time limit of 600 seconds (average CPU time ranging between 0.011 and 313 seconds) and a limit on the number of subproblems set to 4000. With the tolerance level increased to 0.05, only 9 problems (all in the Progen set) remained unsolved. The algorithm was also shown to outperform the procedure by Yang et al. (1992), which could only solve 10 problems, exceeding the CPU time limit for the remaining 80 problems.

4.2.2 Suboptimal approaches

Russell (1986) was the first to test the performance of heuristics for the resource-constrained max-npv problem. He assumes activity-on-the-arc networks taking an event-based view assuming positive and negative cash flows associated with the network events. He conducted an experiment in which six heuristic scheduling rules were tested on 80 different problems. One heuristic is the random rule, which is used as a benchmark (selecting the best out of 50 randomly generated solutions). Two heuristics (the minimum slack rule and the minimum latest finishing time rule) were retained mainly because of their success on the resource-constrained min-duration problem. The remaining three heuristics are based on the optimal results for the unconstrained max-npv problem. As to be expected, no single heuristic performed best on all problems. On small problems, it made little difference which heuristic was used, with the random rule as the best performer. The well-known minimum slack rule performed best on the large-scale problems when the resource-constraints were not tight. For large-scale problems with tight resource constraints, the minimum slack rule was outperformed by one of the three heuristics based on the unconstrained cash flow analysis information.

Smith-Daniels and Aquilano (1987) considered the resource-constrained max-npv problem assuming activity-on-the-node networks where cash outflows occur at the *beginning* of each activity and a single lump-sum payment (cash inflow equal to the cost of the activities plus a percent) is received at the completion of the project. Using an extensive set of 550 test problems generated from the original 110 Patterson problem set, the authors reach the almost self-evident conclusion that a heuristically determined right-shifted schedule (derived from an early-start schedule by right-shifting the activities subject to resource constraints) yields a higher *npv* and lower average duration than schedules derived with heuristics that schedule each activity as early as possible. In addition, while the late-start schedule, on average, was significantly longer than the optimum-duration resource-constrained schedule, a negligible difference occurred in the average *npv*'s of the two scheduling methods.

Baroum and Patterson (1993) propose heuristics which are based on the notion of a cash flow weight (CFW). The CFW of an activity is based on the sum of the cash flows of that activity plus the cash flows of all the activities that must logically follow it in the project. This rule is clearly inspired by the ranked positional weight heuristic originally developed by Helgeson and Birnie (1961) for solving the single-model assembly line balancing problem. The simple forward-pass heuristic selects from the list of available activities the one with the largest CFW (tie-break on the basis of minimum slack, maximum cash flow) and attempts to assign it to the earliest possible completion period without violating precedence and resource constraints. The basic procedure is enhanced with a shifting procedure which shifts activities with a negative cash flow to the right and activities with positive cash flows to the left (see the above simple logic for the unconstrained max-npv problem). The activities are considered in reverse lexicographic order, causing the need for multiple passes. A lower numbered activity that is shifted right might indeed potentially release resources to a higher numbered, but parallel activity, making it possible to right-shift it. The multi-pass right-shifting procedure is followed by a forward, multi-pass left-shift routine which attempts to advance positive cash flow activities into periods in which resources have been incremented by the amounts released from right-shifting activities. The shifting alternates between right-shift and left-shift alternatives until two subsequent iterations result in identical activity assignments. The heuristic may be enhanced by adding discount factors to the undiscounted cash flows when determining the CFWs to be used in establishing activity priority. The authors also develop a *biased* cash flow weight heuristic which uses the cash flow weights of the activities for biasing, selecting the solution with the highest *npv* from 50 problem solutions. The probability for selecting an activity is proportional to its cash flow weight in relation to the cash flow weight of all activities available for scheduling during each scheduling interval. Using a battery of 1540 test problems derived from the 110 Patterson problems, they reach the conclusion that the average *npv* of the CFW heuristic is consistently higher than that of the minimum slack heuristic. None of the single pass heuristics significantly outperform the multi-pass rules. It is interesting to observe that for the resource-constrained min-duration problem, Demeulemeester and Herroelen (1995) have reached the conclusion that their *truncated DH-procedure* without backtracking is competitive to the minimum-slack rule and when allowed to run for a small amount of time (0.01 second) already outperforms multi-pass biased sampling. Also *Pinder* (1988) uses the CFW-approach and suggests several modifications to it.

Focusing on activity-on-the-arc networks with event associated cash flows, *Padman, Smith-Daniels and Smith-Daniels* (1990) use information (revised dual prices and scheduled activity start dates) from the unconstrained max-npv procedure of Russell (1970) in a series of greedy heuristics embedded within a single-pass forward algorithm. These heuristics delay the release of an activity to the queue of schedulable activities until the activities' target schedule date derived from the unconstrained updated network flow solution to the problem becomes current, regardless of when the activities became precedence feasible. In extensive tests utilizing a variety of experimental factors, including project size, progress payment frequency, profit level, and resource utilization, they find that a number of their optimization-guided heuristics provided higher *npv* performance than those included in the study by Russell (1986).

Inspired by research on the job-shop scheduling problem, *Padman and Smith-Daniels* (1993a) hypothesize that releasing activities to the schedule queue as soon as they are precedence feasible may lead to improved *npv* performance, by reducing potential bottlenecks induced by resource conflicts. The heuristics require for each activity the evaluation of the earliness costs and tardiness penalties that are provided by the relaxed optimization model. They test 8 heuristics embedded within a greedy algorithm, similar to the one developed by Padman et al. (1990), on 1440 projects including different network structures, levels of resource constrainedness, and cash flow parameters. They reach the conclusion that the early release heuristics provide superior *npv* results in many project environments.

Sepil and Ortaç (1995) extend the above mentioned model by Sepil and Kazaz (1994) for the unconstrained max-npv problem to the resource-constrained case. They develop three heuristic priority rules. The first rule always gives priority to the activity with the highest *npv*. For all pairs i and j of eligible activities that can be scheduled, the pairwise present value comparison (PPVC) rule computes the *npv* for the case when activity i is scheduled first and vice versa and schedules the activity that provides the highest pairwise *npv*. The activity profit curve slope (APCS) heuristic uses the slopes of the profit curves and gives priority to those eligible activities that if delayed would result with a lower *npv*. They then compare the results with three benchmarking rules (greatest resource demand, least total float and shortest imminent operation).

Ulusoy and Özdamar (1994a) aim at maximizing the *npv* and minimizing the project tardiness in activity-on-the-arc networks subject to a due date where cash inflows and outflows are associated with the events. They develop 6 hybrid rules which are essentially weighted combinations of dynamic slack time and the sum of the *npv* of cash flows on succeeding events. The hybrid rules are embedded in a multipass iterative scheduling algorithm which makes forward and backward scheduling passes, updating the activity time windows at each iteration through the activity start times obtained in the previous schedule. These updated values are used for defining priorities for the activities in the next scheduling iteration. The proposed rules are compared with previously published *npv* and tardiness priority rules and are shown to be superior on a modified subset of the Patterson test problems.

Ulusoy and Özdamar (1995) also apply this type of iterative scheduling procedure on the combined objectives of minimizing project makespan and maximizing *npv*. The iterative scheduling routine is now equipped with four heuristic rules and tested on 78 problems taken from Christofides et al. (1987). In a first problem setting renewable resource constraints are used with activity related cash flows occurring at the start times of activities in combination with a single lump sum payment occurring at project completion. Activity costs depend on the total resource demand of the activity (dollars per unit resource consumption per period). The second setting is a multi-mode environment where each activity has more than one operation mode each representing a different resource-duration trade-off.

Zhu and Padman (1993) study the resource-constrained max-npv in 1440 activity-on-the-arc networks and apply neural networks to induce the relationship between various problem parameters and the performance of 16 heuristic priority rules. They have experimented on different preprocessing schemes and representations of inputs and outputs in the neural net. They have also experimented with a single generalization network approach and a multiple generalization networks approach. The advantage of using the former is that it is simpler and it proposes several heuristics to choose among a few categories. It cannot predict, however, which one of the heuristics in that particular category to use. Multiple generalization network systems in contrast, allow to choose a specific heuristic to use. In general, all of the neural network models perform no worse than multivariate regression models and discriminant analysis.

Padman and Zhu (1994) propose a problem space computational model that integrates the multiple knowledge sources associated with the resource-constrained max-npv problem in activity-on-the-arc networks with event associated cash flows. The model provides a representation (i.e. a hierarchy of problem spaces) within which the various knowledge sources (optimal and suboptimal procedures) are brought to bear in an integrated manner. The knowledge-level model provides an implementation-independent description of the system in terms of its goals (e.g. max-npv), the inputs (e.g. activity-on-the-arc network with associated duration and cash flow data), possible actions (e.g. generate a precedence and resource feasible schedule), the knowledge (e.g. mathematical models and algorithms, heuristics, selection methods), and the environment (e.g. interaction with users and problem solvers). The authors use the model and selection methods developed in Padman et al. (1990) and Zhu and Padman (1993) in order to illustrate the knowledge-level view and formulate the problem space computational model.

Icmeli and Erengüç (1994) study the resource-constrained max-npv problem in activity-on-the-node networks using similar assumptions as Yang et al. (1992); i.e. the objective function of Eq. [6] and the first term of Eq. [5] to compute the value of an activity upon completion. They present a simple single-pass heuristic to determine a starting solution and project due date and develop a tabu search procedure. Given a solution, a move is defined as completing an activity one time unit late or early with respect to its current completion time. The completion time resulting from a move must be between the earliest and latest completion times of the activity under consideration. A move may take the search to an infeasible solution in which case a penalty is computed. The purpose of this penalty is to force the feasible solutions to be selected as the best solutions, and if there exists no feasible solution resulting from the moves in the set of admissible non-tabu moves, the infeasible solution with the least constraint violation is selected as the best solution. The best move is an admissible move which takes the search to a solution with the largest evaluation function value. If the current best move takes the search to a solution that has a better evaluation function value than all the solutions generated in the previous iterations, then this solution is kept as the best solution found so far. If the maximum number of iterations have been performed, the procedure stops with the best solution found. The tabu list keeps record of the attributes of the moves which may lead to re-visiting previously generated solutions. The authors also develop a second procedure which uses a long-term memory function. Computational results on 50 problems generated from the Patterson problem set indicate that the Fortran coded tabu search procedure with the long-term memory functions yields promising results within 10 % of optimality in CPU times ranging from 6 to 46 seconds on an IBM 3090-600J computer.

Yang et al. (1995), clearly inspired by the previous conclusion by Baroum and Patterson (1993) that multi-pass biased sampling procedures perform better than single-pass rules, use 1440 test problems (each consisting of 20 activities) to test 9 so-called stochastic rules. Eight of these rules use single-pass procedures (random, total quantity of resources, sum of cash flows, etc.) to compute the scheduling weight of each activity. This weight governs the probability that the activity is assigned next from the available list of activities. Each of the 8 rules is used to generate 100 solutions for each project, and the best of the 100 solutions is reported. The last heuristic is a simulated annealing procedure. During the generation of a new sample solution this procedure computes the probability that activity j is selected from the available list from the weight $(exp\text{-}\Delta E_j/T)$ assigned to each activity. T is the current temperature while the energy level of activity j is defined as $[1+|jv\text{-}JB_j|]$, where jv records the ordinal position to be occupied next in the sample solution and JB_j records the ordinal position occupied by activity j in the incumbent solution. The energy level of an eligible activity j, ΔE_j, is low if its incumbent ordinal position, JB_j, is close to jv. Conversely, the energy level is high if its incumbent ordinal position is far away from jv. At each fixed temperature T, the scheduling weight assigned to an eligible activity corresponds inversely to its energy level. Therefore, the activity with the incumbent ordinal position closest to the current ordinal position to be occupied has the smallest energy level and the heaviest weight of being scheduled next from the available list. The procedures are coded in Fortran to run on an IBM 3090 mainframe. It was found that the simulated annealing procedure generated the largest number of best and second best *npv* solutions. However, in the presence of infrequent progress payments, bonus receipts and tightly constrained resources, the simulated annealing procedure does not perform as well.

4.3 The resource-constrained max-npv problem with nonrenewable and doubly-constrained resources

Doersch and Patterson (1977) were the first to study capital as a limited non-renewable resource in the context of the resource-constrained max-npv problem. They study the model using the objective given in Eq. [6] and add to the renewable resource constraints Eq. [4] a constraint set which considers the use of capital in each time period. The first term of each constraint indicates that the amount invested in all the activities which can be active during a period must not be greater than the capital available. Specifically it requires that the sum of investments necessary to finish each job in a time period which is less than its duration in the future must not exceed the capital available. The second term of each constraint specifies that the capital available is dependent upon the activities previously scheduled. This term sums, for each activity finished in the past or scheduled to finish less than its duration in the future, the cash flows (not discounted) over that portion of the activity which has been completed. The third term in the constraint indicates that sufficient capital must be available to pay any penalties imposed if the project is completed in the period under consideration. Doersch and Patterson (1977) report on the successful solution of capital-constrained problems consisting of 15 to 20 activities per project using a general purpose integer programming code, although projects consisting of more than 30 activities frequently cannot be solved in a reasonable amount of time. Detailed computational results, however, are not provided.

Smith-Daniels and Smith-Daniels (1987) present a zero-one formulation allowing materials cost and constraints to be added to the basic capital-constrained model of Doersch and Patterson. They discount cash flows to the *beginning* of the activity. Material ordering costs are treated as expenses, are not considered a deduction from capital available and are incurred at the time that an order is received. Inventory holding costs are assumed to be incurred on the ending inventory of the material at the end of each period and are assumed to reduce available capital. Extra constraints are needed to express the inventory balance for each material and each project, and to guarantee that the total capital required by activities, inventories, or performance penalties in any period may not exceed the total capital available at the start of the project. Illustrating the application of the model on a small problem example, the authors do not present a formalized procedure.

Patterson et al. (1990) report on computational experience with a general-purpose backtracking algorithm for solving various types of resource-constrained project scheduling problems. The algorithm has the capability to solve problems with a *max-npv* objective. The algorithm can accommodate capital availability constraints and allows multiple modes for performing an activity where each mode results in a distinct activity and cash flow. The computational results using Fortran code on an IBM 4321 on 91 computer generated problems show that the ability of the algorithm to obtain and verify an optimal solution decreases rapidly once the *npv* objective is introduced. No detailed computational report is given of the percentage of problems solved to optimality, or the size of the largest problem solved, under the *max-npv* objective.

Padman & Smith-Daniels (1993b) apply their optimization-guided approach (Padman and Smith-Daniels 1993a) to the capital-constrained max-npv problem in activity-on-the-arc networks. They assume cash outflows to occur at the start node of an activity (that is, cash outflows are associated with the start of activities) and cash inflows (progress payments) to occur at activity ending nodes (that is, progress payments are associated with the completion of one or more activities). They introduce dummy activities in the activity-on-the-arc network to guarantee that each positive and negative cash flow associated with an activity has a unique node in the network. Capital usage is assumed to occur at a constant level throughout the duration of an activity. The capital balance is increased upon the receipt of cash progress payments. Activities are not released for scheduling until they have reached the target schedule date in Russell's (1970) unconstrained solution. Results on 60 different project network problems with Fortran codes on a Vax computer show promising results for the optimization-guided approach in comparison to the cash flow weight heuristic of Baroum and Patterson (1993).

4.4 Other max-npv models

Baroum (1992) presents a single comprehensive mathematical programming model which allows for renewable resource constraints, capital constraints, borrowing facilities, overhead costs, penalty costs and multiple-modes for accomplishing each activity. As far as we know, no computational results are available on this comprehensive model.

Ulusoy and Özdamar (1994b) present a general framework for an interactive project scheduling system under limited resources. The modeling module allows the decision maker to develop a model with various features such as general cash flow patterns related to the realization of activities or events, progress payments distributed over the project makespan, renewable, nonrenewable and/or doubly-constrained resources, and multiple activity modes. The performance criteria include min-duration, max-npv, and minimum maximal tardiness. The scheduler is based on their forward parallel scheduling local constraint based analysis heuristic which selects activity start times and operating modes on the principle of preserving the feasibility of the schedule with respect to the constraints imposed on the scheduling process by the resource and network features. The authors provide a numerical example and report on a practical MRP installation project.

Erengüç et al. (1991) present a Generalized Benders decomposition procedure for the unconstrained time/cost trade-off problem under the max-npv objective. Their algorithm was tested on 56 test problems, which were all solved to optimality with modest computational cost.

Recently, *Buss and Rosenblatt* (1995) have reported on preliminary results on the stochastic resource-constrained max-npv problem, using an activity-on-the-arc setting, exponential activity durations, negative event cash flows and a single positive cash inflow on project completion.

4.5 The payment scheduling problem

The models discussed so far assume that the timing and amount of the cash flows are known. *Dayanand and Padman* (1993a) argue that the contractor usually knows the expenses associated with project activities but the amount and timing of progress payments are important variables that can be negotiated to improve financial returns. They present five event-based and activity-based models for deciding on the simultaneous determination of the amount and timing of progress payments in the unconstrained max-npv environment. The models assume that (i) the expenses to be incurred over the duration of the project are known, (ii) the contractor incurs expenses more often than he receives payments, (iii) a fixed number of progress payments occur over the duration of the project and are based on the total expenses incurred for the project and a profit margin (cost-plus), and (iv) the contractor estimates the total amount to be received from the owner. This total amount remains unchanged as the project progresses.

The authors derive two event-oriented models departing from the formulation for the unconstrained max-npv problem given by Russell (1970). The models are illustrated on a small problem example. In the first model expenses are incurred at the beginning of activities in the project. Since expenses are associated with events and several activities may commence at an event, expenses at any event are the total expenses incurred for all activities commencing at that event. Progress payments are also associated with events. In addition to 0-1 variables associating cash flows with events, the first model represents the occurrence times by continuous variables; in the second model binary variables are used to define occurrence times as well as assignment of cash flows to events. The authors also derive a third event-oriented model based on an extension of Grinold's (1972) model. One of the limitations of these event-based models is that they assume that some expenses have to be incurred much earlier than necessary to have the project completed on schedule. In addition, total expenses at every event are obtained by aggregating expenses for all activities commencing at that event. The authors present a modified event model in which every real activity is preceded by a dummy activity. The tail node of the dummy activity is the starting node for every real activity. Expenses are associated with these nodes. Since payments are associated with the completion of activities, they are associated only with tail nodes of real activities. The authors also present an activity-based model, clearly inspired by the formulation in Yang et al. (1992), which assumes activity-on-the-node networks, known expenses associated with each activity and payments made at the completion of activities.

Dayanand and Padman (1993b) subsequently refined the second event-based model by reducing the number of integer variables and by indexing progress payments by time; i.e. each time period has a progress payment associated with it. The actual number of payments is restricted by constraining K of these variables to be greater than zero. They also present and test several simple heuristics on two data sets derived from Russell (1986) and Padman & Smith-Daniels (1993b).

5. Conclusions and suggestions for further research

The orthodox theory of capital budgeting advocates the *npv* rule for making accept/reject decisions on investment projects. The crucial assumption made by the various researchers on the project scheduling problem with discounted cash flows is that the *npv* rule should either be used in project *bidding* (*tendering*) or in *scheduling* the project activities once the project is accepted. When faced with preparing a bid to submit to the owner of a project, the contractor must analyze the contract and project specifications. The assumption made is that this analysis includes the formulation of a project network, the preparation of the time estimates for the project activities and the determination of the resources that are required to complete it. In a *stochastic* situation (such as e.g. in PERT type of networks) the realization time of network events as well as the costs of activities can be determined only in a probabilistic sense. Bidding procedures in such an environment have been studied by Elmaghraby (1990). In a *deterministic* situation with known activity durations and costs, the streams of cash flows (cash outflows, representing the payments made by the contractor and cash inflows representing the receipts by the contractor) can then be obtained *given a particular schedule of the activities*. The issue then resembles the case of projects already accepted for execution and revolves around the *scheduling of activities to maximize the net present value of the project*. It is clear that the relevance of the max-*npv* objective is limited to those situations where the time value of money may come into play, that is in capital intensive projects with a sufficient long duration (several months or even years), significant cash flows, high interest rates and high cost of capital. Large construction projects such as typically found in the building industry constitute a typical environment. The *use of the max-npv objective in the (finite) scheduling of production activities* (machine scheduling problems such as the job-shop, see Morton et al. 1988, Morton and Pentico 1933) *or service activities over a relatively short period of time* (hours, days, weeks) *can be seriously questioned*. This conclusion seems to be confirmed by the findings of Scudder and Smith-Daniels (1989) who state that the differences in *npv* between the dispatching rules tested in their experiment are relatively small for even medium length jobs (operation processing times varied between .50 and 18 hours).

Where applicable, it is important that the assumptions made by the researchers are in line with the specifications of the various contract types and terms of payment. The research efforts on the *deterministic max-npv problem* assume a single project for which the cash flows are deterministically known in both their amount and timing. In addition all models assume finish-start precedence constraints with a time lag of zero and no task-preemption. Despite its basic formulation as a nonlinear programming problem, the basic solution rationale underlying the *unconstrained max-npv* is simple: advance as much as possible the positive cash flows, delay as much as possible the negative cash flows. Almost all the optimal procedures developed in the literature take an event-oriented view where the cash flows are associated with the events in activity-on-the-arc networks. In the absence of a project due date, the optimal schedule may be to delay the project for ever, and under certain conditions it may be the case that any schedule is optimal. The solution methodology for the event-based models using activity-on-the-arc networks can be extended to an activity-oriented view using activity-on-the-node models. Efficient computer codes are available for the optimal procedures developed by Russell (1970), Grinold (1972) and Elmaghraby and Herroelen (1990). As a result, the deterministic unconstrained max-npv problem may be considered as efficiently solved. No research on heuristic procedures is needed.

The *stochastic unconstrained max-npv problem* is a very hard nut to crack. Research is just in its infancy. The single research effort made so far assumes negative *exponentially* distributed activity durations and presents optimal and suboptimal procedures for determining the optimal amount of activities' delay, beyond their earliest start times, so as to maximize the *expected* present value of the project. *Under conditions of uncertainty, however, there is a fundamental reason to question the use of the max-npv rule.* The *npv* rule advocated by the orthodox capital budgeting theory assumes that either an investment is reversible or, if the investment is irreversible, it is a now or never proposition. However, the ability to delay an irreversible investment expenditure undermines the simple *npv* rule. The reason is that a firm with an opportunity cost to invest is holding an "option". When a firm makes an irreversible investment expenditure, it kills its option to invest by giving up the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure. This lost option value is an opportunity cost that must be included as part of the cost of the investment. As a result, the *npv* rule must be changed. Dixit and Pindyck (1994) discuss how optimal investment rules can be obtained from methods that have been developed for pricing options in financial markets. They illustrate how this theory can be applied to sequential investment decisions in multistage projects. A promising research effort lies in the extension of these option theory-based arguments to the resource-constrained project scheduling environment.

Optimal procedures for the *deterministic resource-constrained max-npv problem* take an activity-oriented view. Most models allow for progress payments and assume activity cash flows to be compounded to the end of the activity. The best results have been obtained using a depth-first branch-and-bound procedure (Icmeli and Erengüç 1995) inspired by the optimal DH-procedure (Demeulemeester and Herroelen 1992) which was developed for solving the min-duration problem. The computational efficiency reported on projects with less than 50 activities and 3 resource types, however, is not of the quality which has been recently reported for the min-duration problem (Demeulemeester and Herroelen 1995). As for the min-duration objective, computational improvement might be obtained using efficient 32-bit coding and stronger bounding and/or dominance arguments.

A wide variety of *heuristics* for both the event-oriented and activity-oriented view on resource-constrained max-npv problems has been reported in the literature. It seems that the abundant research efforts on studying the performance of priority dispatching rules under the min-duration objective are repeated for the max-npv objective. In addition to single-pass heuristics, a number of efforts aim at the development of multi-pass procedures based on biased sampling. Recent efforts have also applied local search heuristic methodology (tabu search and simulated annealing). The heuristics have been tested on problem sets which vary widely in their assumptions and the reported results are not very conclusive. A comprehensive experiment testing the most promising procedures under identical problem assumptions (project network structure, cash flows, types of resource constraints) on a standard set of test problems is still missing.

Research on the *payment scheduling problem* which aims at determining both the timing and amount of cash flow payments has just emerged. It is obvious that this type of problem setting is limited to those situations where the contract specifications allow for decisions on timing and amount of payments. Event-oriented and activity-oriented formulations have been presented. A limited set of heuristic test results is available.

As shown in this paper, the research efforts on the project scheduling problem with discounted cash flows are numerous and of an impressive variety in both problem assumptions and solution methodologies. Most research efforts assume a deterministic problem setting in which activity durations, cash flows and discount rate are known from the outset. In a way, one might argue that these efforts hold a kind of contradiction. On one hand, the use of the *max-npv* criterion in a short-term scheduling environment can be seriously questioned. On the other hand, large-scale capital intensive projects that span a long period of time (years) essentially constitute a stochastic problem setting in which the deterministic assumptions no longer hold. Under such conditions of uncertainty, however, recent findings in the area of investment analysis under uncertainty reflect a more fundamental reason to question the use of the max-npv rule. Attempts to exploit the recent findings and option-based methodologies developed as part of a new emerging theory of investment under uncertainty may shed some new light on the complex field of project scheduling.

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